X non-compacte, ou

$$\int_X |\omega| \qquad \omega \ \text{parfois s'annule.}$$

$|\omega|$ pas loc. cste.

$$N_m = \frac{1}{p^{mn}} \# \{ x \in (\mathbb{Z}/p^m\mathbb{Z})^n \mid f(x) = 0 \}$$

$$P_f(t) = P(t) = \sum_{i \geq 0} N_m t^m \ ? \qquad f \in \mathbb{Z}[x]$$

t est petit

$0 \leq N_m \leq 1$.

$$Z_f(s) = \int_{x \in \mathbb{Z}_p^n} |f(x)|^s \cdot |dx| \qquad s \geq 0.$$

$p^{-\operatorname{ord} f(x) \cdot s}$ zeta $\left\{ \begin{array}{l} \text{locale} \\ \text{d'Igusa.} \end{array} \right.$

$$P(t) = \frac{1 - t \cdot Z_f(s)}{1 - t} \qquad t = p^{-s}.$$

$$N_m = \mu(\ldots) = \mu_{\text{Haar}}(\ldots)$$

$$= \mu \underbrace{\{ x \in \mathbb{Z}_p^n \mid \operatorname{ord} f(x) \geq m \}}_{V_m}.$$

$$= \mu(V_m) \qquad x \in (\mathbb{Z}/p^m)^n$$

$$Z_f(s) = \sum_{i \geq 0} p^{-si} \int_{\mathrm{ord}\, f(x) = i} |dx| \quad V_i \setminus V_{i+1}$$

$$= \sum_{i \geq 0} p^{-si} \left(\mu(V_i) - \mu(V_{i+1}) \right)$$

$$= \sum_{i \geq 0} p^{-si} \left(N_i - N_{i+1} \right)$$

$$= \underbrace{\sum_{i \geq 0} p^{-si} N_i}_{p^{-s} = t} - p^{s} \left(\sum_{i \geq 1} N_i \, p^{-si} \right)$$

$$= \underbrace{\sum_{i \geq 0} t^i N_i}_{P(t)} - p^{s} \left(P(t) - 1 \right)$$

$$Z_f(s) = P(t) - t \left(P(t) - 1 \right)$$

$$\underbrace{P(t)} = \frac{1 - t \, Z_f(s)}{1 - t}$$

$$\int_{\mathbb{Z}_p^n} |f(x)|^s \, |dx| \longrightarrow \log \cup p^{\mathbb{Z} \cdot s} \qquad s > 0 \text{ fixé}$$

$\mathbb{Z}_p^n$ — composite

$f(x) = 0$?

exemple simple

$$f(x) = \text{monôme} = \prod_i x_i^{e_i}$$

$$\int_{\mathbb{Z}_p} |x| \cdot |dx| = \frac{(1-p^{-1})}{\cdots}$$

$$Z_f(s) = \prod_{i=1}^{n} \underbrace{\int_{\mathbb{Z}_p} |x_i^{e_i}|^s |dx|}$$

(Fubini)

$$Z_{x^e}(s) = \int_{x \in \mathbb{Z}_p} |x|^{es} |dx| = \sum_{i \geq 0} p^{-ies} \int_{\underbrace{\operatorname{ord} x = i}} |dx|$$

$$\bigcup_{a=1}^{p-1} \underbrace{a p^i + p^{i+1}\mathbb{Z}_p}$$

$$= \sum_{i \geq 0} p^{-ies} \frac{(p-1)p^{-i-1}}{\cancel{\quad}} = \frac{p-1}{p^{-1}} \underbrace{\sum_{i \geq 0} p^{-i(es+1)}}_{\text{géom}}$$

$$= \frac{p-1}{p^{-1}} \cdot \frac{1}{1-p^{-es-1}}$$

$$t = p^{-s}$$

$$= c \cdot \frac{1}{1-p^{-1} \cdot t}$$

$$= \frac{c}{1-dt}$$

rationelle en t

$Z(s)$ analytique en $t = p^{-s}$

$\in \mathbb{Q}(t)$

? $Z_f(s)$ nature géométrique

? rationnelle en t ?

? cas monômes utiles ?

$$\int_{\mathbb{Z}_p^2} |x_1 + x_2^2|^s \, |dx| \Rightarrow$$

mon: rat.